Sección 3-3

ejercicios ~~1~~, ~~3~~, ~~5~~, ~~7~~, ~~9~~, 11, 13, 15, 17.

① $x(0) = x_0$

$$\frac{dy}{dt} + \lambda_2 y = \lambda_1 x_0 e^{-\lambda_1 t}$$

$$\frac{d}{dt}\left[e^{\lambda_2 t} y\right] = \lambda_1 x_0 e^{(\lambda_2 - \lambda_1)t} + C_2$$

$$y = \frac{\lambda_1 x_0}{\lambda_2 - \lambda_1} e^{(\lambda_2 - \lambda_1)t} e^{-\lambda_2 t} + C_2 e^{-\lambda_2 t}$$

$$y = \frac{\lambda_1 x_0}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} + C_2 e^{-\lambda_2 t}$$

$y(0) = 0$

$$y = \frac{\lambda_1 x_0}{\lambda_2 - \lambda_1} \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

$$\frac{dz}{dt} = \frac{\lambda_1 \lambda_2 x_0}{\lambda_2 - \lambda_1} \left(e^{-\lambda_1 t} - e^{-\lambda_1 t}\right)$$

$$z = \frac{\lambda_2 x_0}{\lambda_2 - \lambda_1} e^{-\lambda_1 t} + \frac{\lambda_1 x_0}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} + C_3$$

$z(0) = 0$

$$z = x\left(1 - \frac{\lambda_2}{\lambda_2 - \lambda_1} e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t}\right)$$

③ las cantidades de x y y son las mismas aproximadamente t=5 días. las de x y z son aproximadamente t= 20 días. las cantidades de y y z son las mismas aproximadamente t = 14.7 días.

El tiempo cuando y y z es el mismo, y tiene sentido porque la mayor parte de A y la mitad de B se han ido, por lo que la mitad de C debería haberse formado.

⑤ $$x'_1 = 2 \cdot 3 + \frac{1}{50} x_2 - \frac{1}{50} x_1 \cdot 4$$

$$= -\frac{2}{25} x_1 + \frac{1}{50} x_2 + 6$$

$$x'_2 = \frac{1}{50} x_1 \cdot 4 - \frac{1}{50} x_2 - \frac{1}{50} x_2 \cdot 3$$

$$= \frac{2}{25} x_1 - \frac{2}{25} x_2$$

⑦ a) Un modelo es

$$\frac{dx_1}{dt} = 3 \cdot \frac{x_2}{100 - t} - 2 \frac{x_1}{100 + t} \qquad x_1(0) = 100$$

$$\frac{dx_2}{dt} = 2 \cdot \frac{x_1}{100 + t} - 3 \frac{x_2}{100 - t} \qquad x_2(0) = 50$$

b) Como el sistema es cerrado, $x_1(t) + x_2(t) = 100 + 50$

$x_1 = 150 - x_2$

$$\frac{dx_2}{dt} = \frac{2(150 - x_2)}{100 + t} - \frac{3x_2}{100 - t}$$

$$= \frac{300}{100 + t} - \frac{2x_2}{100 + t} - \frac{3x_2}{100 - t}$$

$$\frac{dx_2}{dt} + \left(\frac{2}{100 + t} + \frac{3}{100 + t} \right) x_2 = \frac{300}{100 + t}$$

$\Rightarrow$

$$e^{2\ln(100 + t) - 3\ln(100 - t)} = (100 + t)^2 (100 - t)^{-3}$$

$$\frac{d}{dt} (100 + t)^2 (100 - t)^{-3} = 300 (100 + t)(100 - t)^{-3}$$

$$(100 + t)^2 (100 - t)^{-3} x_2 = 300 \left[\frac{1}{2} (100 + t)(100 - t)^{-2} - \frac{1}{2} (100 - t)^{-1} + C \right]$$

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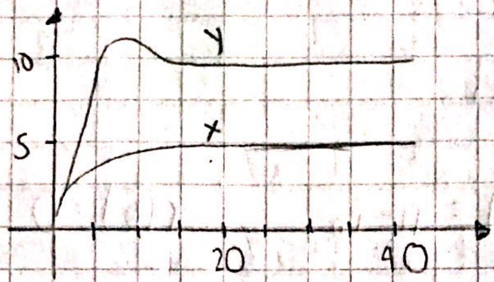
$$X_2 = \frac{300}{(100+t)^2}\left[e^{(100-t)^3} + t(100-t)\right]$$

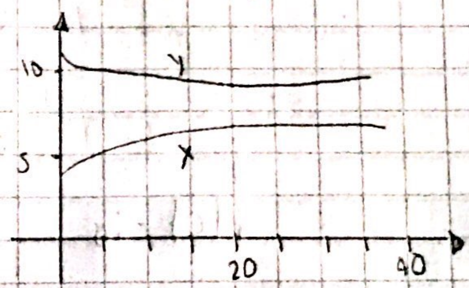
$$X_2(0) = 50$$

$$C = \frac{5}{3000} \qquad t = 30 \qquad X_2 = \left(\frac{300}{130^2}\right)\left(70^3 c + 210\right)$$

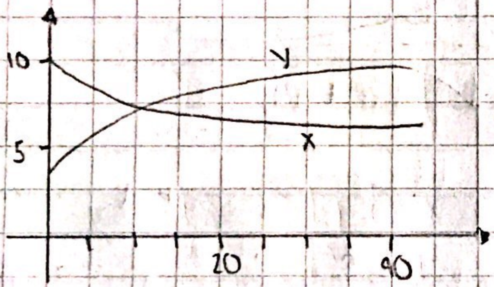
(11) a)



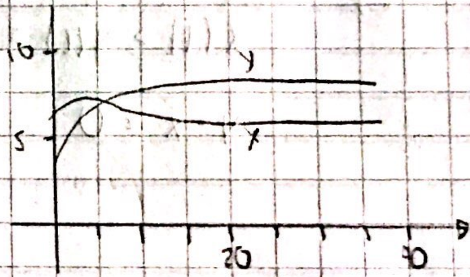
b)



c)



d)



x(t) enfoca 6,000
y(t) enfoca 8,000.

(13)

$$E(t) = i_1 R_1 + L_1 \frac{di_2}{dt} + i_2 R_2$$

$$E(t) = i_1 R_1 + L_2 \frac{di_3}{dt} + i_3 R_3$$

$$L_1 = \frac{di_2}{dt} + (R_1 + R_2)\, i_2 + R_1 i_3 = E$$

$$L_2 = \frac{di_3}{dt} + R_1 i_2 + (R_1 + R_3)\, i_3 = E$$

(15) $$\frac{d}{dt}(s+i+r) = \frac{d}{dt}n = 0$$

$$\frac{di}{dt} = -\frac{dr}{dt} - \frac{ds}{dt} = -k_2 i + k_1 s i$$

$$\frac{ds}{dt} = -k_1 s i$$

$$\frac{di}{dt} = -k_2 i + k_1 s i$$

$$\frac{dr}{dt} = k_2 i$$

$i(0) = i_0$ $\qquad s(0) = n - i_0$ $\qquad r(0) = 0$

(17) $x_0 > y_0 > 0$

$x(t) > y(t)$

$y - x < 0$